6-7 Escalonadas
7-8
8-9 Narrativas Naturais (ou seguinte)

O que torna as narrativas possíveis

Naturais: A estrutura que se precisa ter para ter as narrativas

Espaços Topológicos - ~~Est~~ Estruturas que se precisa ter para ter as funções contínuas.

Espaços Vetoriais - Estrutura que se precisa ter para ter as aplicações lineares.

livre $(\mathbb{N}, 0, s)$ livre

E_0 G a, b —— 0 com 0 gerador

E_1 a', b' —— $0'$ "$\mathbb{N}$

E_2 a'', b'' —— $0''$

$\{0\}$ $0' = 0$

ordem

Categoria de estruturas $\{0,1\}$ $0' = 1$ $1' = 0$ / $0' = 0$ $1' = 1$

$h: A \to B$ morfismo

$h(0) = 0$

$h(s(a)) = s(h(a))$

$h: \mathbb{N} \to A$ sobrejetora

$h(0) = 0$ 1) injetora $\mathbb{N} \simeq A$

2) Seja m o 1º $\in \mathbb{N}$ tal que $h(m) = h(n)$ com $m \neq n$

$h(0) = 0$ — $h(m-1)$ distintos

Pilot ~~P~~ Pilot Pilot

$0 \quad 0' \quad 0''$ —— $0^{(n)}$ —— livre com

$a \quad a' \quad a''$ —— $a^{(n)}$ —— a gerador

$0 \equiv a$ op. const. $\mathbb{N}$ [diagrama de estr. livre

duas operações

E_0 ~~$0 \; 0' \; 0''$~~ 0 gerador 0

E_1 $0' \quad {}'0$ $s(0) \quad t(0)$

$0'' \quad ''0 \quad '(0') \quad ('0)'$ $ss(0) \quad tt(0) \quad st(0) \quad ts(0)$

$0''' \quad '''0 \quad ('(0'))'$

—— x ——

Totalmente ordenado Bem ordenado

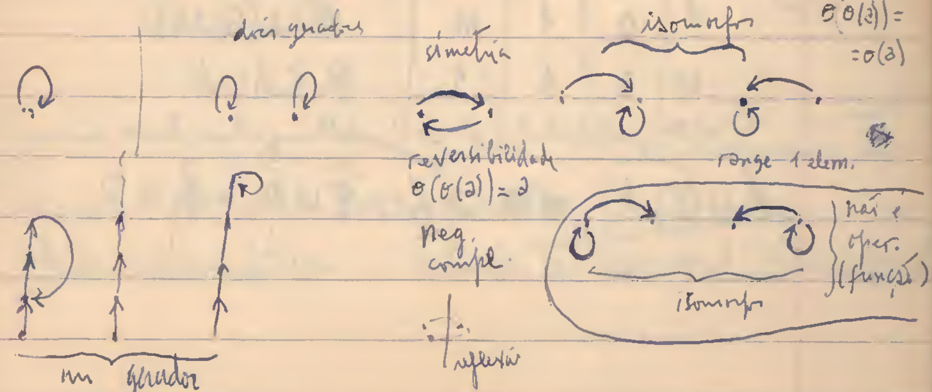
—— x ——

Endomorfismo $h: \mathbb{N} \to \mathbb{N}$ (não nec. pres. 0)

$h(a') = h(a)'$ movimento

$b + 0 = b$ $h(0) = b$

$b + a' = (b+a)'$ $h(a') = h(a)'$



dois geradores 

$a+a \qquad a+(a+a) = (a+a)+a$

Tomando um ponto de vista construtivo "construímos" tanto o conjunto quanto a operação.

1) Dado um conjunto, "construir" a operação

2) Construir o conjunto e, simultaneamente, a operação. (Ou a operação "gera" o conjunto)

$\mathring{A} \subset \overset{\circ}{\widehat{A\cup B}}$ $\quad$ 22 $\quad$ 4,5

$\mathring{B} \subset \overset{\circ}{\widehat{A\cup B}}$ $\quad$ 22

$\mathring{A}\cup\mathring{B} \subset \overset{\circ}{\widehat{A\cup B}}$ $\quad$ 90

90

990

Se $A = \mathbb{R}$

e $A \supset B = \mathbb{R}$

$\overset{\circ}{\widehat{R' \cup B}} = \mathring{B} = \mathbb{R}$

$B = \mathbb{R}$

$\nearrow$	0	1	m
0	1	1	1
1	0	1	m
m	0	1	1

$A \supset (B \supset A)$

$\overset{\circ}{\widehat{A' \cup B' \cup A}}$

$\mathring{A} \subset \overset{\circ}{\widehat{A\cup B'}} \Rightarrow A \subset \overset{\circ}{\widehat{B' \cup A}}$

$A' \cup (B' \cup A)$

$\mathbb{R} \subset A \cup A'$

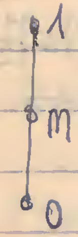
$\overset{\circ}{\widehat{A' \cup (\overset{\circ}{\widehat{B' \cup A}})}} \supset \overset{\circ}{\widehat{A' \cup B' \cup A}} = \mathring{\mathbb{R}} = \mathbb{R}$

$A = (-\infty, 0) \cup (0, \infty)$

$$\underbrace{(\underbrace{(\underbrace{A \supset \emptyset}_{\emptyset}) \supset A}_{\mathbb{R}}) \supset A}_{A} = A \qquad \mathring{A}' = \emptyset \qquad A \neq \mathbb{R}$$

$\mathring{A}' = \{a\} \qquad a \in \mathbb{R}$

$A = \{a\}'$



$$\underbrace{(\underbrace{(\underbrace{m \supset 0}_{0}) \supset m}_{1}) \supset m}_{m} = m$$

6ª 18

5ª 17

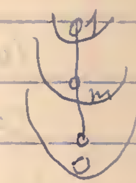
4ª 15

3ª 14

2ª 13

$(a \supset b) \supset b \qquad a \vee b$

$E = \{0, 1, m\}$ Fechados $\{1\}, \{1, m\}, E$



Teoria Matemática Intuitiva — Estuda conjuntos, funções, números, etc.

Teoria Matemática Formalizada

Metamatemática — Estuda matematicamente a Teor. Mat. Formalizada

$\mathring{A} \subset \widehat{A \cup B}$ $\quad \mathring{A} \cup \mathring{B} \subset \widehat{A \cup B}$ $\qquad a \vee (a \supset b)$

$\mathring{B} \subset \widehat{A \cup B}$ $\quad \{ F ((a \supset b) \supset a) \supset a \}$

$A \supset B = \phi$ $\quad \{ T (a \supset b) \supset a, \ Fa \}$

$\widehat{A' \cup B} = \phi$ $\quad \{ F a \supset b, \ , Fa \} \quad \{ Ta, Fa \}$

$\{ Ta, \ Fb \}$

$((\mathring{a} \supset 0) \supset a) \supset a$

$((0,1) \cup (1,2)) \supset (\frac{1}{2}, \frac{3}{2}) = \cancel{(-\infty,0] \cup [1,2]}$

$= (-\infty,0] \cup \{1\} \cup [2,+\infty) \cup (\frac{1}{2}, \frac{3}{2})$

$\cancel{(-\infty,0)}$

$= (-\infty,0) \cup (1/2, 3/2) \cup (2,\infty)$ $\quad 0 \ \tfrac{1}{2} \ 1 \ \tfrac{3}{2} \ 2$

$((-\infty,0) \cup (1/2,3/2) \cup (2,\infty)) \supset (0,1) \cup (1,2) =$

$\widehat{[0,1/2] \cup [3/2,2] \cup (0,1) \cup (1,2)} = (0,1) \cup (1,2)$

$((0,1) \cup (1,2)) \supset$

$(0,1) \supset \phi = [-\infty,0] \cup [1,+\infty) = [0,1]'$

$[0,1]' \supset (0,1) = \widehat{[0,1] \cup (0,1)} = (0,1)$

$\bigcup\limits_{n \in \mathbb{Z}} (n,n+1) \supset B = \widehat{\mathbb{Z} \cup B}$ $\qquad \phi \supset B = \widehat{E \cup B} = E$

$E \supset B = \widehat{\phi \cup B} = B$

$((-\infty,0) \cup (0,+\infty)) \supset \phi = \widehat{\{0\} \cup \phi} = \phi$

$\phi \supset ((-\infty,0) \cup (0,+\infty)) = \mathbb{R}$

$\mathbb{R} \supset ((-\infty,0) \cup (0,+\infty)) = (-\infty,0) \cup (0,+\infty)$

$((a \supset b) \supset a) \supset a$

$(0,1) \supset (1,2) = \widehat{(0,1)' \cup (1,2)} = \widehat{(-\infty,0] \cup [1,+\infty) \cup (1,2)}$

$= (-\infty,0) \cup (1,\infty) = [0,1]'$

$[0,1]' \supset (0,1) = \widehat{[0,1] \cup (0,1)}$

$0 \ 1 \ 2 \ 3$

$(0,2) \supset (1,3) = \widehat{(0,2)' \cup (1,3)} = \widehat{(-\infty,0] \cup [2,+\infty) \cup (1,3)}$

$= \widehat{[-\infty,0] \cup (1,+\infty)} = (-\infty,0) \cup (1,+\infty) = [$

$[0,1]' \supset (0,2) = \widehat{[0,1] \cup (0,2)} = \widehat{[0,2)} = (0,2)$

$(0,2) \supset (0,2) = \widehat{(0,2)' \cup (0,2)} = \mathring{\mathbb{R}} = \mathbb{R}$

$(-\infty,0) \supset (0,+\infty) = \widehat{(-\infty,0)' \cup (0,+\infty)} = (0,+\infty)$

$(0,+\infty) \supset (-\infty,0) = \widehat{(0,+\infty)' \cup (-\infty,0)} = (-\infty,0)$

$(-\infty,0) \supset (1,+\infty) = \widehat{(-\infty,0)' \cup (1,+\infty)} = (0,+\infty)$

$\phi \supset (0,1) = \widehat{\phi' \cup (0,1)} = \mathbb{R}$

$\mathbb{R} \supset \phi = \widehat{\mathbb{R}' \cup \phi} = \phi$

$(0,1) \supset (0,2) = \widehat{(0,1)' \cup (0,2)} = \widehat{[-\infty,0] \cup [1,+\infty) \cup (0}$

$\mathbb{R} \supset (0,1) = \widehat{\mathbb{R}' \cup [-\infty,0] \cup [1,\infty)} = (0,1)$

$(0,2) \supset (0,1) = \widehat{(0,2)' \cup (0,1)} = \widehat{[-\infty,0] \cup [2,+\infty) \cup (0,1)}$

$= (-\infty,1) \cup (2,+\infty) = [1,2]'$

$[1,2]' \supset (0,2) = \widehat{[1,2] \cup (0,2)} = (0,2)$

$S, T(x\supset y)$ $\quad$ $S, F(x\supset y)$

$S, FX \mid S\,TY$ $\quad$ S_T, TX, FY

$\{F((a\supset b)\supset(b\supset a))\supset(b\supset a)\}$

$\{T(a\supset b)\supset(b\supset a),\ F\, b\supset a\}$

$\{T(a\supset b)\supset(b\supset a),\ Tb,\ Fa\}$

$\cancel{\{F(a\supset b), Tb, Fa\}}$

$\cancel{\{Fa, Tb\}}$ $\{F a\supset b, Tb, Fa\}$ $\{T\, b\supset a, Tb\ Fa\}$

$\cancel{\{Tb, Ta, Fb\}\ \{T\, b\supset a, Tb, Fa\}}$

$\cancel{\{Tb, Ta, Fb\}\ \{Fb, Fa, Tb\}}$

$\{Ta, Fb, Tb\}$ $\quad$ $\cancel{\{Tb, Fa, Tb\}}$

$\{Fb, Ta, Tb\}$

$(A\supset(B\supset C))\supset((A\supset B)\supset(A\supset C))$

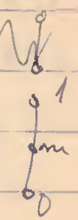
$A \to a\supset b$

$B \to b$

$C \to a$

$((a\supset b)\supset(b\supset a))\supset(((a\supset b)\supset b)\supset((a\supset b)\supset a))$

$\cancel{(a\supset a)\supset a}$

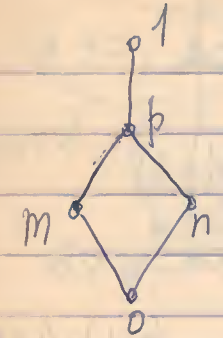


$a\supset((a\supset a)\supset a)$

$a\supset((a\supset b)\supset b)$

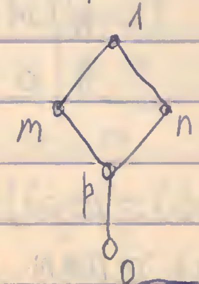
1 1 1 m m 1 m

m m 1 1



$m\supset p = 1$

$p\supset m$



$F((a\supset b)\supset(b\supset a))\supset(b\supset a)$

$T(a\supset b)\supset(b\supset a)$ $\quad$ $F(b\supset a)$

$T(a\supset b)\supset(b\supset a)$ $\quad$ Tb $\quad$ Fa

$F(a\supset b)$	$T(b\supset a)$
$Ta\ Fb$	

$a\le b \Rightarrow a\supset b = 1$

$\to b\supset a$

$b\wedge(b\supset a)\le a$

$b\wedge x\le a \Rightarrow x\le b\supset a$

$(((a\supset b)\supset b)\supset((a\supset b)\supset a))\supset(b\supset a)$

$((a\supset b)\supset a)\supset a$

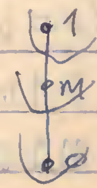
m 1 m m

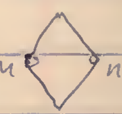
m n n m m 1 m

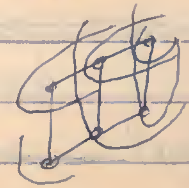
→ m 0 0 m m 1 m

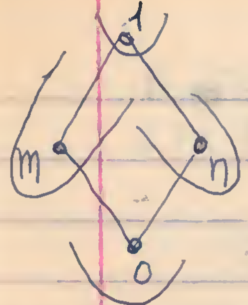
0 1 m 0 0 1 0

m









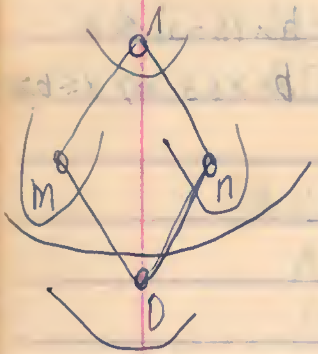
$\supset$	0	m	n	1
0	1	1	1	1
m	n	1	n	1
n	m	m	1	1
1	0	m	n	1

$((a\supset b)\supset(b\supset a))\supset(b\supset a)$ $\qquad (a\supset b)\supset(b\supset a),\ b$

mnn mnmm 1 nmm

nmmnmnn 1 mnn

mn01mn0 1 01m

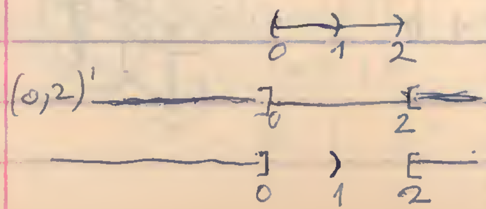


$\supset$	0	m	n	1
0	1	1	1	1
m	0	1	n	1
n	0	m	1	1
1	0	m	n	1

$((a\supset b)\supset(b\supset a))\supset(b\supset a)$

0 1 m 0 m00 1 m00

m0 0 1 01m 1 01m



$(0,1)\subseteq(0,2)$

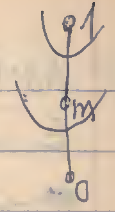
$(0,2)\supset(0,1)$

$\overline{(0,2)'\cup(0,1)}$

$(\infty,1)\cup(2,\infty)=[1,2]'$

a, b

$(a\supset a,\ b\supset b),\ a\supset b,\ b\supset a$



$(a\supset b)\vee(b\supset a)$

$((a\supset b)\supset(b\supset a))\supset(b\supset a)$

m1m $\quad$ 1mm $\quad$ 1m

~~$a\supset b,\ b\supset a$~~

1 $\quad$ m $\quad$ m

$\supset$	0	m	1
0	1	1	1
m	0	1	1
1	0	m	1

0 m 1 $\quad$ m 0 0 $\quad$ m 0 0 $\quad$ 0

0 x 0 $\quad$ 0 x m $\quad$ 0 1 $\quad$ 0 m

m 0 0 $\quad$ m 1 m $\quad$ m 1 1

1 0 0 $\quad$ 1 m m $\quad$ 1 1 1

$((a\supset b)\supset(b\supset a))\supset(b\supset a)$

010 1 010 1 010

1001 011 1 011

0100 100 1 100

01m0m00 1 m00

m00 001m 1 01m

m11m 1mm 1 1mm

1mm1 m11 1 m11

$C A\cup B$

$\mathring{A'}\cup B \qquad \overline{A'\cup B}$

$((a\supset b)\supset a)\supset a$

$A\supset A$

$(\overline{A'\cup B})\cup A \qquad (0,1) \qquad \mathring{A'}\cup A$

$a\vee\sim a \qquad A\cup\mathring{A'} \qquad \overline{A'\cup A}$

a $(0,1)$ $\qquad (0,1)'=(\infty,0]\cup[1,\infty)$ $\qquad$ b $\emptyset$

$a\supset b \qquad (\infty,0)\cup(1,\infty)$

Narrativas

E $\quad \mathcal{S}$ conjunto das seqüências finitas de elem. de E.

~~S é uma narr.~~ $\quad p : S \longrightarrow \mathcal{P}(E)$

narrativa $s \in S$ tal que

$$s(0) \in p(\Lambda)$$
$$s(1) \in p($$
$$s(n) \in p(s(0) \ldots s(n-1))$$

a	$a \vee b$	
$(a \supset a) \supset a$	$(a \supset b) \supset b$	$a \supset (b \supset b)$
V V V V V	V V V V V	V V V V V
F V F F F	V F F V F	V V F V F
	F V V V V	F V V V V
	F V F F F	F V F V F

$(a \supset (b \supset c)) \supset ((a \supset b) \supset (a \supset c))$

$(a \supset b) \supset (a \supset c), a, b \vdash b \supset (a \supset b)$

$\vdash a \supset b$

$\vdash a \supset c$

$\vdash c$

$(a \supset b) \supset (a \supset c), a \vdash b \supset c$

$(a \supset b) \supset (a \supset c) \vdash a \supset (b \supset c)$

$b \in \bar{A} \quad \longrightarrow b \in \overline{\{a_1, \ldots, a_n\}} \quad a_i \in \bar{A}$ tipo finito

$\longrightarrow b \in \overline{\{a_1 \ldots a_{n-1}\} \cup \{a_n\}}$

$\Longrightarrow a_n \supset b \in \overline{\{a_1, \ldots, a_{n-1}\}}$

$\longrightarrow a_{n-1} \supset (a_n \supset b) \in \overline{\{a_1 \ldots a_{n-2}\}}$

$\ldots$

$\Longrightarrow (a_1 \supset (a_2 \supset (\ldots (a_n \supset b)) \ldots) \in \Phi$

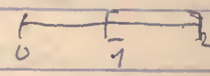
$b \in \bar{A} \Leftrightarrow$ existe uma seqüência finita $a_1, \ldots, a_n = b$ tal que $a_i \in \Phi$ ou $a_i \in A$ ou $a_j \supset a_i = a_k$ com $j, k < i$

$a_k \in \bar{A} \quad a_j \supset a_i \in \bar{A} \quad a_j \in \bar{A}$

$\{a_j \supset a_i, a_j\} \in \bar{A}$

$a_i \in \bar{A}$

Em estrutura algébrica a passagem de A para $\bar{A}$ (aplicando as operações) como uma dedução (analysis) (completamente) (imanente) (implícita)



$(0,2) \supset (0,1) = [1,2]'$

$[1,2]' \supset (0,2) = [1,2] \cup (0,2) = (0,2)$

~~$x \in \mathbb{Z}$ que somados a 1~~

$\{x \in \mathbb{Z} : x+1 \leq 3\}$

$\{\ldots -n, \ldots, -1, 0, 1, 2\}$ segmento inferior ≤ 2

tem $2 = 3-1$ como maior elemento

$\{n \in \mathbb{Z} : x+3 \leq 1\}$

$\{\ldots, -n, \ldots, -4, -3, -2\}$ seg. inf. ≤ -2

tem $-2 = 1-3$ como maior elemento

$a + \overset{-a+b}{(b-a)} \leq b$

Se $a + x \leq b$

$a + x - a \leq b - a$ compatibilidade

$x \leq b - a$

Logo $b-a$ é o maior x tal que $a + x \leq b$

(Introdução da diferença)

(com ênfase em $\leq$)

$a + (-a+b) \leq b$

Se $a + x \leq b$ então $x \leq -a+b$

$a \wedge x \leq b$

Se $a \wedge y \leq b$ então $y \leq x$

Tomemos $x = a' \vee b$

$a \wedge (a' \vee b) = \underbrace{(a \wedge a')}_{0} \vee (a \wedge b) \leq b$

Se $a \wedge y \leq b$

$(a \wedge y) \vee a' \leq b \vee a'$

$(a \vee a') \wedge (y \vee a') \leq b \vee a'$ xxx

$y \leq y \vee a' \leq b \vee a'$

reticulado distributivo com último elemento, dados a, b existe c tal $a \wedge c \leq b$

se $a \wedge x \leq b$ então $x \leq c$

$0,010101\ldots$

$\frac{0}{2^1} + \frac{1}{2^2} + \frac{0}{2^3} + \frac{1}{2^4} + \ldots$

$\frac{1}{2^1}$; $\frac{0}{2^1} + \frac{1}{2^2} = \frac{1}{4}$, $\frac{2}{8}$, $\frac{4+1}{16}$

$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots = \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = 1/3$

2^2 2^4

$(a \supset (b \supset c)) \supset ((a \supset b) \supset (a \supset c))$

$a \to b$ é o conjunto dos $c \in E$

tais que

$b \in \overline{\{a, c\}} = \overline{\{a\} \cup \{c\}}$

Se $b \in \overline{\{a\} \cup A}$ então $\overline{\{b\}} \subseteq \bar{A}$

suponhamos c ∃

$b \in \overline{\{a, b\}} = \overline{\{b\} \cup \{a\}}$

15:30 $\overline{\{b\}} \subset \overline{\{a\}}$ $\overline{\{c\}} \subset \overline{\{a\}}$

$c \in \overline{\{a\}}$

Se $c \in a \Rightarrow b$ então $c \in \overline{\{a\}}$

$a \equiv b$ se $\overline{\{a\}} = \overline{\{b\}}$

$c, d \in \cancel{a \to b}$ $a \supset b$

$b \in \overline{\{a, c\}}$ $b \in \overline{\{a, d\}}$

Se $b \in \overline{\{a\} \cup \{d\}}$ então $\overline{\{c\}} \subset \overline{\{d\}}$ se $b \in \overline{\{a\} \cup \{c\}}$ então $\overline{\{d\}} \subset \overline{\{c\}}$

Logo $\overline{\{c\}} = \overline{\{d\}}$

$A' \cup B$

$a + (b - a) = b$

$\begin{cases} a + (-a + b) = b \\ a + x = b \text{ então } x = -a + b \end{cases}$

O maior x tal que $a + x \leq b$

O maior x tal que $a \cdot x \leq b$

O maior X tal que $A \cap X \subseteq B$