# Instituto de Investigaciones Filosóficas
# (IDIF)

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Dear Professor Lawvere,

Thank you very, very much for sending to me the commentaries on your contribution to foundational research in the Cambridge meeting. Reading it has been very illuminating to me on several important themes.

When I visited you in Buffalo I was developing some ideas about the relationship between logic and mathematics which I mentioned to you, but only in a preliminary form because, at that time, I had not a complete clear idea about the whole matter. Now I have reached some conclusions which, I think. can be of interest for foundational research.

The main idea is the following. As you know, beside intuitionistic - logic, a different and complementary logic has been deviced under the name of paraconsistent logic. In intuitionistic logic, the principle of the excluded third, cannot be derived.

Because of that it receives the general name of paracomplete, that is, two contradictory propositions can be false without trivializing the system. In paraconsistent logic, it is the principle of contradiction that can not be derived.

New if topoi are models for paracomplete logics, there must be cate - gorical structures that are models for paraconsistent logics. I call these structures "Khoroi". A khoros must have opposite and complementary properties of topoi. So, in a topos $f \cap -f=0$ but not necessarily $f \cup -f=1$; in a khoros $f \cup -f=1$, but nos necessarily $f \cap -f=0$.

This clearly shows that there are models of a paraconsistent logical system that cannot --- be topoi and that a structure, to be a model of a paraconsistent system must satisfy the condition $f \cup -f=1$. And there must be also categorical structures that satisfy the proposi - tion $p \,\&\, \neg p$.

How must be these structures characterized in categorical language?. I think they must have some common traits with topoi. For instance - they must have a classifier, because if they don't they cannot be - models of logical systems. They must also have exponentiation, because

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if they don't then it will be impossible to develop set theory. N.C.A. da Costa has developeda really interesting inconsistent set theory by using paraconsistent logic. It is a theory in which all the theorems of classical set theory can be derived and in which it is impossible to derive theorems that contradict thems but in which it is possible, without trivializing the system, to introduce Russell's set. This introduction leads to wonderful theorems, completely diferent from the classical ones. In spite that this set theory is simply inconsistenty is absolutely consistent.

We see, that a khoros must have two common traits with a topos: exponentation and classifier. So the only possibility of differentation lies in the completeness and cocompleteness condition. Let's supose that a khoros is not finitely complete (nor cocomplete). This means a serious difficulty because completeness is fundamental to establish the existence of pullbacks.

Another possibility would be to supose that a khoros is infinitely complete and cocomplete. But this would be a too strong condition and, besides, it would make topoi particular cases of a khoroi.

As you see I'm in a quandary. The other possibility would be to define a khoros as a categorical structure with classifier, exponientiation and incomplete (and coincomplete) and to give up the use of the pullback as the fundamental concept of the theory. Of course the new concept ought to comply with the condition of establishing universal properties.

But, be it as may be, there is no doubt that there must exist categorical structures that are models of paraconsistent logics N.C.A. da Costa and his followers have ellaborated some algebraic systems that are, in fact, models of some extant paraconsistent logics. But these models are not expressed in categorical language nor have been deviced to be compared with the topos structure.

I know you are awfully busy but perhaps you could devote a few moments to consider the points I have indicated. I would very much appreciate if you could give me some hints on the line of thought that could be followed to define the concept of khoros.

With warms regards,

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P.S.: "Khoros" means in greek the same as "topos". But there is an important difference. Whereas "topos" has only a spatial meaning, "khoros" can have besides its spatial meaning, a hyerarchical meaning. To say in greek that a person occupies the first place in honour, you say that he occupies the first khoros, and not the first topos. In paracomplete logic two contradictory propositions can be false, but not true; and in paraconsistent logic two contradictory propositions can be true (but not false). And truth, beeing a higher epistemological value than falsity, paraconsistency has a higher cognitive value than paracompleteness. Of course this is only metaphorical, and means absolutely nothing from a scientific point of view. But its nice, isn't it?

FMQC/amh.

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